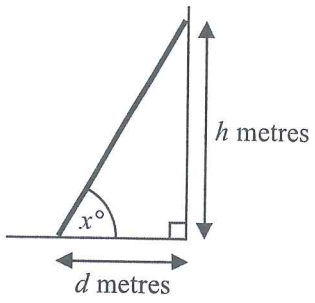
11 The bottom of a ladder is on horizontal ground.
The top of the ladder is leaning against a vertical wall.

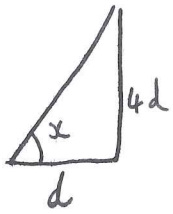


The bottom of the ladder is d metres from the wall.
The top of the ladder is h metres above the ground.
The angle between the ladder and the ground is $x°$

Some safety instructions say it is safe to climb the ladder when

$$h = 4d$$

(a) Work out the value of x when $h = 4d$



$\tan x = \frac{4d}{d}$ $\tan x = 4$ $x = \tan^{-1}(4)$

$x = 75.96375653$

76°

(3)

Some different safety instructions say the angle between the ladder and the ground should be 75°
The ladder is moved so that $x = 75$

(b) How does this affect the height, h metres, of the top of the ladder above the ground?

Decreases h for the same value of d

(1)

(Total for Question 11 is 4 marks)

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12 Here are the first four terms of a quadratic sequence.

3 8 15 24

(a) Find an expression, in terms of n, for the nth term of this sequence.

① 3 8 15 24

5 7 9

2 2

$2a=2$

$a=1$

n^2	1	4	9	16
Subtract	2	4	6	8

2 2 2

$2n$

n^2+2n

② 3 8 15 24

5 7 9

$3a+b=5$

$5a+b=7$

$a=1$, $3(1)+b=5$

$b=2$

$a+b+c=3$

$1+2+c=3$

$c=0$

n^2+2n

(3)

The nth term of a different sequence is 2^n+5

(b) Show that 36 is **not** a term of this sequence.

$2^n+5=36$

$2^n=31$, since n is an integer 31 is not a power of 2

(1)

(Total for Question 12 is 4 marks)

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13 Alex wants to find out how many ducks there are in a park.

One day he puts a tag on each of 30 of the ducks.
The next day he catches 40 ducks.
8 of these ducks have tags on them.

(i) Work out an estimate for the number of ducks in the park.

$40 \div 8 \times 30 = 150$

or $\frac{8}{40} = \frac{30}{n}$ $\quad n = \frac{40 \times 30}{8} = 150$

Alex assumed that none of the tags fell off during the night.

(ii) If Alex's assumption is wrong, explain how this could affect your answer to part (i).

Then effectively the sample of 30 is reduced leading to an over estimate

(Total for Question 13 is 4 marks)

14 Given that $3^{-n} = 0.2$

find the value of $(3^4)^n$

$(3^4)^n = 3^{4n}$

$3^{-n} = \frac{1}{5}$

$3^{-n} = 5^{-1}$

$(3^{-n})^{-4} = (5^{-1})^{-4}$

$3^{4n} = 5^4 = 625$

(Total for Question 14 is 2 marks)

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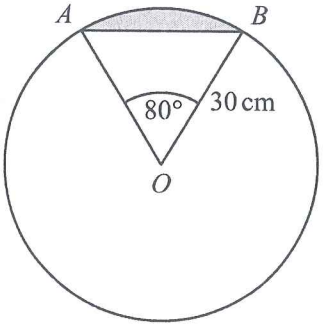
15



AB is a chord of a circle centre *O*.

The radius of the circle is 30 cm.
Angle *AOB* = 80°

Work out what percentage of the area of the circle is shaded.

Area of Sector $= \frac{80}{360} \times \pi 30^2 = 200\pi$

Area of $\triangle = \frac{1}{2}(30)(30)\sin 80 = 443.16346664$

Area of Shaded Segment $= 200\pi - 443.16\ldots$
$= 165.1550419$

Area of Shaded Segment as % of circle

$\frac{165.1550419}{\pi \times 30^2} \times 100 = 6.548520034$

6.55 ..... %

(Total for Question 15 is 5 marks)

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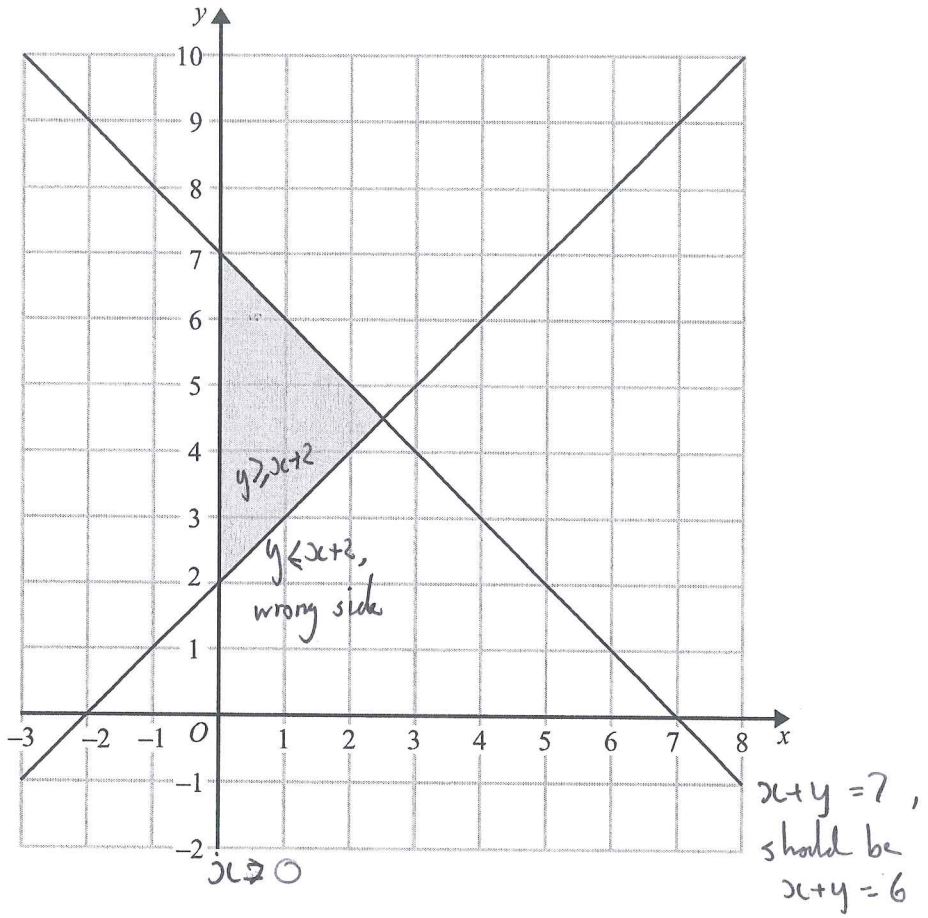
16 For her maths homework, Helen answered the following question.

Shade the region that is defined by all these inequalities.

$x + y \leqslant 6$ $\quad y \geqslant 0$ $\quad y \leqslant x + 2$

Here is Helen's answer.



Helen made some mistakes when she answered the question.

Write down two mistakes Helen made.

1 The line is $x+y=7$, not $x+y=6$

2 Area under $y=x+2$ is required, not the area above

(Total for Question 16 is 2 marks)

(3) line $x=0$ used instead of $y=0$

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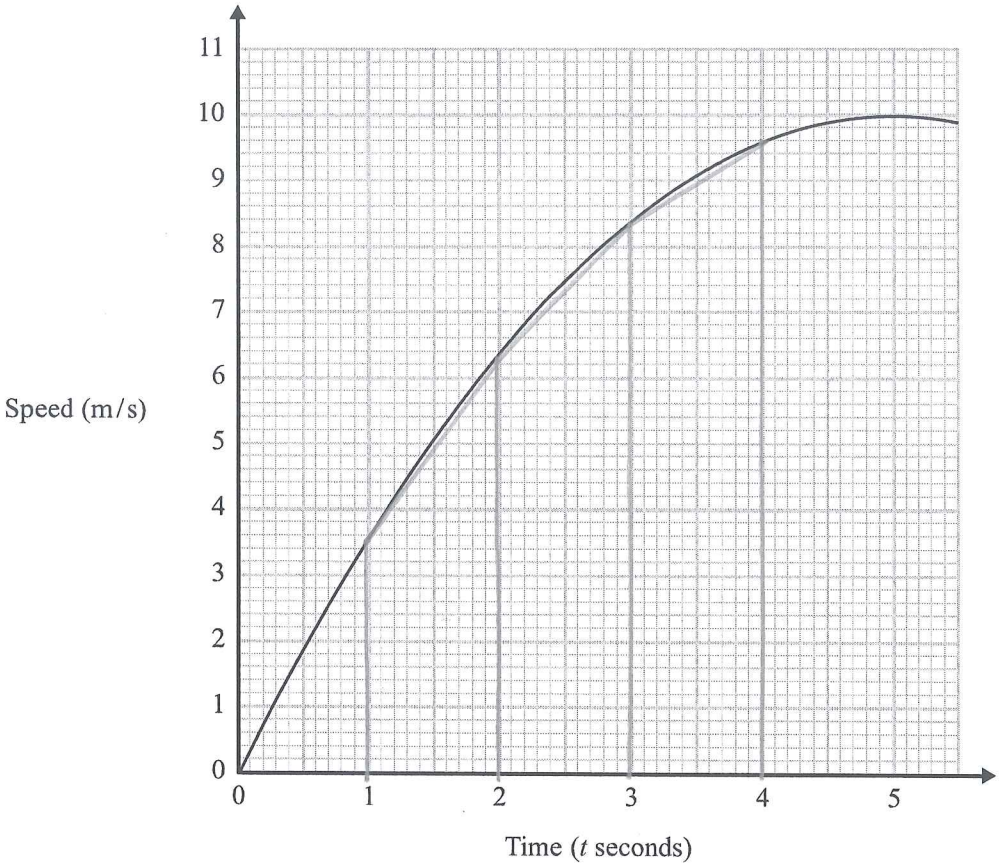
17 Here is a speed-time graph showing the speed, in metres per second, of an object t seconds after it started to move.



(a) Use 3 strips of equal width to find an estimate for the area under the graph between $t = 1$ and $t = 4$

Trapezium Rule

$$\text{Area} = \frac{1}{2}\left(3.6 + 9.6 + 2(6.4 + 8.4)\right)$$

$$= 21.4$$

(3)

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(b) Describe fully what your answer to part (a) represents.

Area under a speed/time graph is distance.

(2)

(c) Explain whether your answer in part (a) gives an underestimate or an overestimate for the area under the graph.

underestimate as the chords are all below the curve

(1)

(Total for Question 17 is 6 marks)

18 There are 95 girls and 87 boys in Year 13 at a school.

One girl is going to be chosen for the role of Head Girl.
A different girl is going to be chosen for the role of Deputy Head Girl.
One boy is going to be chosen for the role of Head Boy.
A different boy is going to be chosen for the role of Deputy Head Boy.

Work out how many different ways this can be done.

$95 \times 94 \times 87 \times 86 = 66814260$

(Total for Question 18 is 3 marks)

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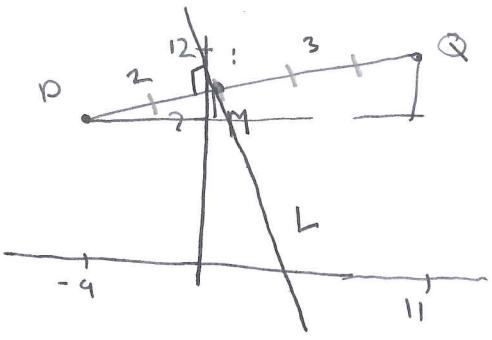
19 P has coordinates $(-9, 7)$
Q has coordinates $(11, 12)$

M is the point on the line segment PQ such that $PM : MQ = 2 : 3$

Line **L** is perpendicular to the line segment PQ.
L passes through M.

Find an equation of **L**.



Gradient $PQ = \frac{12-7}{11-9} = \frac{5}{20} = \frac{1}{4}$

$PM : 2/5$ along from P

$x : -9 + \frac{2}{5}(20) = -1$

$y = 7 + \frac{2}{5}(5) = 9$

$M\ (-1, 9)$

Gradient of $\perp \Rightarrow -4$

$y = -4x + c$

Sub $9 = -4(-1) + c$

$9 = 4 + c$

$c = 5$

$y = -4x + 5$

.....

(Total for Question 19 is 5 marks)

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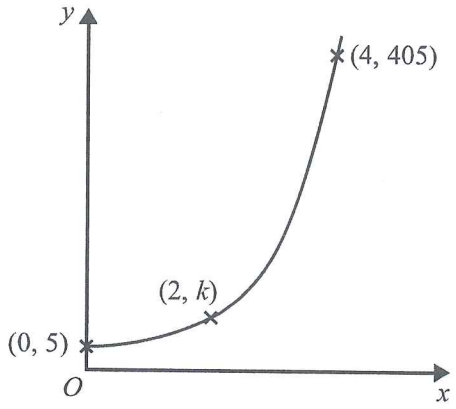
20 Here is a sketch of part of the graph of $y = pq^x$ where $q > 0$



The points (0, 5), (2, k) and (4, 405) are all on the graph of $y = pq^x$

Find the value of k.

When $x = 0$ $y = 5$ $\therefore$ $5 = pq^0$ since $q^0 = 1$

$p = 5$

when $x = 4$ $y = 405$ $\therefore$ $405 = 5q^4$

$q^4 = 81$

$q = \sqrt[4]{81} = 3$

when $x = 2$ $y = 5(3)^2$

$y = 45$

(Total for Question 20 is 4 marks)

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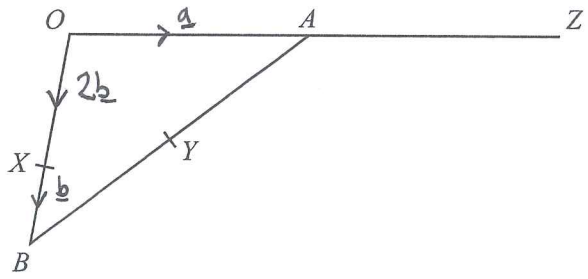
21



OAB is a triangle.
A is the midpoint of *OZ*
Y is the midpoint of *AB*
X is a point on *OB*

$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OX} = 2\mathbf{b}$ $\overrightarrow{XB} = \mathbf{b}$

Prove that *XYZ* is a straight line.

$\overrightarrow{AB} = 3\underline{b} - \underline{a}$

$\overrightarrow{AY} = \frac{1}{2}(\overrightarrow{AB}) = \frac{1}{2}(3\underline{b} - \underline{a})$

$\overrightarrow{XY} = \overrightarrow{XO} + \overrightarrow{OA} + \overrightarrow{AY}$

$= -2\underline{b} + \underline{a} + \frac{1}{2}(3\underline{b} - \underline{a})$

$= -\frac{1}{2}\underline{b} + \frac{1}{2}\underline{a} = \frac{1}{2}(\underline{a} - \underline{b})$

$\overrightarrow{XZ} = \overrightarrow{XO} + \overrightarrow{OZ} = -2\underline{b} + 2\underline{a} = 2(\underline{a} - \underline{b})$

$\overrightarrow{XY}$ & $\overrightarrow{XZ}$ have the point X in common.

Since $\overrightarrow{XY}$ $(\frac{1}{2}(\underline{a} - \underline{b}))$ and $\overrightarrow{XZ}$ $(2(\underline{a} - \underline{b}))$ are scalar multiples of each other, they are parallel.

∴ XYZ is a straight line Q.E.D.

(Total for Question 21 is 5 marks)

TOTAL FOR PAPER IS 80 MARKS

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