**Part 4**

**PROMPT Sheet**

**10/1 Calculating in Standard Form A x 10n**

WITHOUT A CALCULATOR

Addition & subtraction

* change to normal numbers
* Add/subtact
* Convert back to standard form

e.g. 3x102 + 1.8x103 = 300 + 1800 = 2100 = 2.1x103

Multiplication & division

* work out number part
* work out the power of 10 part
* check answer is standard form

e.g. 3 x 105 x 4 x 103 = 12 x 108 = 1.2 x 109

WITH A CALCULATOR

 use or

X10x

EXP

**10/2 Estimate roots and powers**

Example: Estimate the value of 3√70

**13  23 33 43 53**

**1 8 27 64 125**

3√70 lies between 4 and 5 so would be **≈ 4.1**

Example: Estimate **8214**

824 = ( 8.21 x 102)4

 ≈84 x 108

 = 642 x 108

 ≈ 602 x 108

 = 3600 x 108 or 3.6 x 1011

**10/3 Zero/negative/simple fraction indices**

* Multiply & divide

**ax x ay = a(x + y) ax ÷ ay = a(x - y)**

* Raise a power to a power

**(ax)y = a (x y)** (a3)2 = a6  (23)2 = 26 = 64

* Zero index

**a0 = 1**  y0 = 1 80 = 1

* Negative index

**a-x = 1** a-3 = 1 2-3 = 1 = 1

 **ax** a3 23 8

* Fractional index

**a1/2 =** $\sqrt{a}$ **a1/3 =** $\sqrt[3]{a}$ **a1/4 =** $\sqrt[4]{a}$

**ax/y = ()x** a2/5 = ()2 322/5 = ()2 =22

**10/4 Recurring decimals to fractions**

If x = 0.4444444 If x = 0.54545

10x = 4.4444444 100x =54.545454

 9x = 4 99x = 54

 x = 4 x = 54

1. 99

**10/5 Product rule for counting**

If there are ‘**m’** ways of doing one task and for each of these, there are **‘n’** ways of doing another task then the **total number of ways** the two tasks can be done is ‘**mxn’**

Example 1: If a cafe sells 8 different cakes and 6 different drinks, the total number of combinations for a cake and a drink is 8x6 = 48

Example 2: Two letters from the alphabet are chosen but not two the same.

The total number of combinations is 26x25= 650

**10/6 Expand 3 binomials**

Example: (x+5)(x+2)(x-3)

* Multiply the last 2 brackets

(x+5)(x2 - x – 6)

* Multiply all terms in 2nd bracket by x then by 5

x3 - x2 – 6x + 5x2 - 5x – 30

* Collect like terms together

x3 + 4x2 -11x -30

**10/7 Factorise quadratic expressions**

form ax2 + bx + c

Difference of 2 squares

4x2 – 4x – 3 = (2x + 3)(2x -1)

4x2 – 25 = (2x – 5)(2x + 5)

 **Solve quadratic equations-factorising**

* **Put equation in form ax2 + bx + c = 0**

2x2 – 3x – 5=0

* **Factorise the left hand side**

(2x – 5)(x + 1) = 0

* **Equate each factor to zero**

2x – 5 = 0 or x + 1 = 0

 **x = 2.5 or x = -1**

**10/8 Equations of perpendicular lines**

* Two lines are perpendicular if they meet at a right angle (90°)
* The product of their gradients is **-1**

These lines are perpendicular:

y = 4x – 3

Because 4 x **-**$\frac{1}{4}= -1$

y= **-**$\frac{1}{4}$ x ± ?

**10/9 Find equation of line given two points**

* **Graphically**

Example: Plot the two points (0,1) and (1,3)

The equation: y = mx + c



‘m’ is the gradient = 2 ÷ 1 = 2

‘c’ is where the graph intercepts the y-axis: **1**

So equation of line is: y=2x+**1**

* **Algebraically**

Gradient (m) = y2 – y1 = 3-1 = 2

 x2 – x1 1-0

Equation: y - y1 = m (x – x1) OR y = mx + c

 y – 1 = 2(x- 0) (Substitute one of the points)

 y – 1 = 2x

 y = 2x + 1

**10/10 Find equation of line given one point**

** & its gradient**

Example

**Constant velocity**

Gradient=2 and (1, 5)

**~Graphically**

Equation: y = mx ± c

**Constant deceleration**

**Constant acceleration**

 y = 2x ± c

**Area under graph = distance travelled**

y = 2x + 3

**~Algebraically**

**stationary**

Substitute x=1,y=5

5 = 2x1 +c

c=3

Equation: y = 2x + 3

**10/11 Find roots and turning point of**

 **quadratic graphically**

(4, 0)

(0, 0)

(2, -4)

x

y

ROOTS are x=0 and x=4 - points where the graph cuts the x-axis (i.e. where y = 0)

TURNING POINT is (2,-4) - axis of symmetry passes through the turning point

**10/12 Find roots of quadratic function**

 **algebraically**

* Rearrange to the standard quadratic form
* Factorise
* Solve

Example

x2 = 3x + 4

x2 – 3x - 4 = 0 rearranged

(x – 4)(x + 1) = 0 factorised

x-4 = 0 or x+1 = 0 solved

x= 4 or x=-1 roots

**10/13 Velocity-time Graph**

The gradient of a velocity-time graph represents the acceleration

* the **area** under a velocity-time graph represents the **distance** covered
* Horizontal line is **constant velocity**

**10/14 Solve linear inequalities graphically**

First plot the straight lines as equations

Broken line for inequalities.

Decide which side of the line to shade.

Leave the region required unshaded.

e.g. ***x* ≤3 *y* > –2 *y* < *x***



R

**10/15 Calculate nth term of a quadratic sequence**

* Find the 2nd difference, it will be constant
* Halve the 2nd difference to get ... n2

Examples

2nd difference 2, sequence will start with n2

2nd difference 4, sequence will start with 2n2

2nd difference 6, sequence will start with 3n2

* Write down the sequence of ...n2
* Original sequence minus ...n2 sequence
* Find nth term of what is left – linear
* Form nth term using ...n2 and linear in n

**10/16 Geometric sequences**

Each term after the first is found by multiplying the previous one by a fixed number (**common ratio)**

**Geometric sequence are powers of a fixed number – rn**

Examples:  2n – 20, 21, 22, 23, 24,.. **Common ratio = 2**

 3n – 30, 31, 32, 33, 34,.. **Common ratio = 3**

**General form of a geometric sequence is:**

a, ar, ar2, ar3, ar4, ... arn-1 (n is the term number)

(where *r* is the common ratio & *a* is the start value)

* Geometric sequences

 1, 2, 4, 8, 16 ....... **Common ratio = 2**

* Surd sequence

1, √2, 2, 2√2, ..... **Common ratio = √2**

**10/17 Growth & Decay problems**

**A = P(1 ± r)n**

New amount

%rate

as a decimal

Initial amount

Time e.g. months, years etc

To increase £12 by 5% per year for 4 yr

**= £12 x 1.054 (1 + 0.05)**

To decrease £50 by 12% per year for 4 yr

**= £50 x 0.884 (1 – 0.12)**

**10/18 Direct and inverse proportion**

The symbol means:

 ‘varies as’ or ‘is proportional to’

* **Direct proportion**

If: **yx** or **yx2** or **yx3**

Formulae: **y = kx** or **y = kx2** or **y = kx3**

**Example**

y is directly proportional to x

When y = 21, then x = 3

*(find value of k first by substituting these values)*

**yx** y = kx

 21= k x 3

  k = 7

 y = 7x

*(Now this equation can be used to find y, given x)*

* **Inverse proportion**

If: y1 or **y1** or **y1**

 x **x2 x3**

Formulae: **y = k** or **y = k** or **y = k**

  **x**  **x2**  **x3**

**Example**

a is inversely proportional to b

When a = 12 and b = 4

a1 a = k

 b b

 12= k

 4

  k = 48

 a = 48

 b

*(Now this equation can be used to find a, given b)*

**10/19 Standard circle theorems**

The angle at the centre The angle in a semi-circle

= 2 x the angle at the is a right angle

circumference

1800

Opposite angles of a cyclic quadrilateral add up to 1800

Angles in the same

segment are equal

Tangents from a point to a circle are equal. Angle between tangent & radius = 900

The perpendicular from

the centre to a chord

bisects the chord

The angle between a tangent and a chord is equal to the angle in the alternate segment

**10/20 Enlarge by a negative scale factor**

* The image is on the opposite side of the centre
* The image is also inverted

Example : Enlargement scale factor -2 about 0



**10/21 Length of arc**

 sector

 arc

 segment

 r

 θ

**Length of arc** = θ x 2πr

 3600

**10/22 Area of sector**

 r

 θ

**Area of sector** = θ x πr2

 3600

**10/23 Surface area of spheres & pyramids**

**Curved surface area *(formula NOT given)***

Curved surface area of a cylinder = 2πrh

 r

 h

Curved surface of a cone = πrl ***(formula given)***

 h l

 r

*[NB To find ‘l’ use Pythagoras’ Theorem*

 *l 2 = h 2 + r 2]*

Curved surface of a sphere = 4πr2***(formula given)***

 r

**10/24 Volume of spheres & pyramids**

**Volume - pyramid**

Volume of Pyramid =  x area of cross-section x height

e.g. cone

 h

 h

 w

 r l

Volume = x πr2h Volume = x l x w x h

***(formula given)***

**Volume - sphere**

Volume of Sphere = πr3

***(formula given)***

**10/25 & 26 Similarity & enlargement**

* **For similar shapes when:**

**Length** scale factor = **k**

**Area** scale factor = **k2**

**Volume** scale factor = **k3**

Example

 4cm A 6cm B

If height of A = 4cm & height of B = 6cm

* Length scale factor = 6 ÷ 4 = **1.5**

If surface area of A = 132cm2

* Surface area of B = 132 x **1.52** = 297cm3

If volume of A = 120 cm3

* Volume of B = 120 x **1.53** = 405cm3

**10/27 Vectors**

If **a =**$ \left(\genfrac{}{}{0pt}{}{3}{2}\right)$ and **b** = $\left(\genfrac{}{}{0pt}{}{-2}{2}\right)$

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
|  |  | **b** |  |
|  |  |  |  |
|  |  |  |  |
|  | **a** |  |  |

**Addition of vectors**

a + b=$\left(\genfrac{}{}{0pt}{}{3}{2}\right)$+$\left(\genfrac{}{}{0pt}{}{-2}{2}\right)$=$\left(\genfrac{}{}{0pt}{}{1}{4}\right)$

a + b

**Subtraction of vectors**

a - b=$\left(\genfrac{}{}{0pt}{}{3}{2}\right)$ - $\left(\genfrac{}{}{0pt}{}{-2}{2}\right)$=$\left(\genfrac{}{}{0pt}{}{5}{0}\right)$

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  | - **b** |  |
|  | **a**a + b |  |  |  |

**Multiplication by a scalar**

2a =$2\left(\genfrac{}{}{0pt}{}{3}{2}\right)$=$\left(\genfrac{}{}{0pt}{}{6}{4}\right)$

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |
|  |  | 2**a**  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  | **a** |  |  |  |  |

**10/28 Conditional probabilities**

The first event influences the second event

* **TREE DIAGRAM**

Example

2 milk and 8 dark chocolates in a box

Kate chooses one and eats it. (ONLY 9 left now)

She chooses a second one

 **1st** **2nd**  M MM= 2 x 1= **2**

 1 10 9 **90**

 2 M 9

2milk 10 8

8dark 9 D MD= 2 x 8 = **16**

 8 10 9 **90**

 10 2 M DM= 8 x 2 =**16**

 D 9 1 0 9  **90**

 7

 9

 D DD= 8 x 7 = **56**

 10 9  **90**

* **VENN DIAGRAM**

Example

This diagram represents the percentages of a set of people who take part in Athletics & Basketball.

**Basketball**

**Athletics**

43%

39%

18%

**p(A**$|$**B)** is the probability of choosing a person who does Athletics, **given** the person does Basketball =$\frac{43}{82}$ ≈52%

* **TWO-WAY TABLE**

Example

|  |  |  |
| --- | --- | --- |
|  | **Athletics** | **Basketball** |
| **Male** | 17 | 12 |
| **Female** | 14 | 10 |

The probability of choosing a female, given that she plays basket ball = $\frac{10}{22}$

**10/29 Cumulative frequency table & graph**

|  |  |  |  |
| --- | --- | --- | --- |
| Mark | f | Upper limit | cf |
| 0≤x<10 | 4 | <10 | **4** |
| 10≤x<20 | 18 | <20 | 4+18=**22** |
| 20≤x<30 | 30 | <30 | 4+18+30=**52** |
| 30≤x<40 | 72 | <40 | 4+18+30+72=**124** |
| 40≤x<50 | 54 | <50 | 4+18+30+72+54=**178** |
| 50≤x<60 | 22 | <60 | 4+18+30+72+54+22=**200** |

 Median

Upper Quartile

Lower Quartile

Median (M) = 37 marks

Upper quartile (UQ) = 44 marks

Lower quartile (LQ) = 30 marks

Inter-quartile range (IQR) = 44 – 30 = 14 marks

**10/30 Box plots** (for data in 10/29)

 IQR(50% of the data)

 LQ M UQ

**Use of box plots to compare two** **Distributions**



These are used to make comparisons which help to reach a conclusion:

1. **Average** – median in box plots
2. **Spread** – IQR in box plots (width of box)

*NOTE:*

*The bigger the spread the less consistent*

*The range is not used as the measure of spread as it could be affected by one or two outliers*

Conclusion:

The boys on average are taller; the spread of heights of the girls is greater than the boys